RC filters and signal conditioning
MEC100x-Lectures 4

Energy, Power and Intelligent Control

School of Electronics, Electrical Engineering and Computer Science

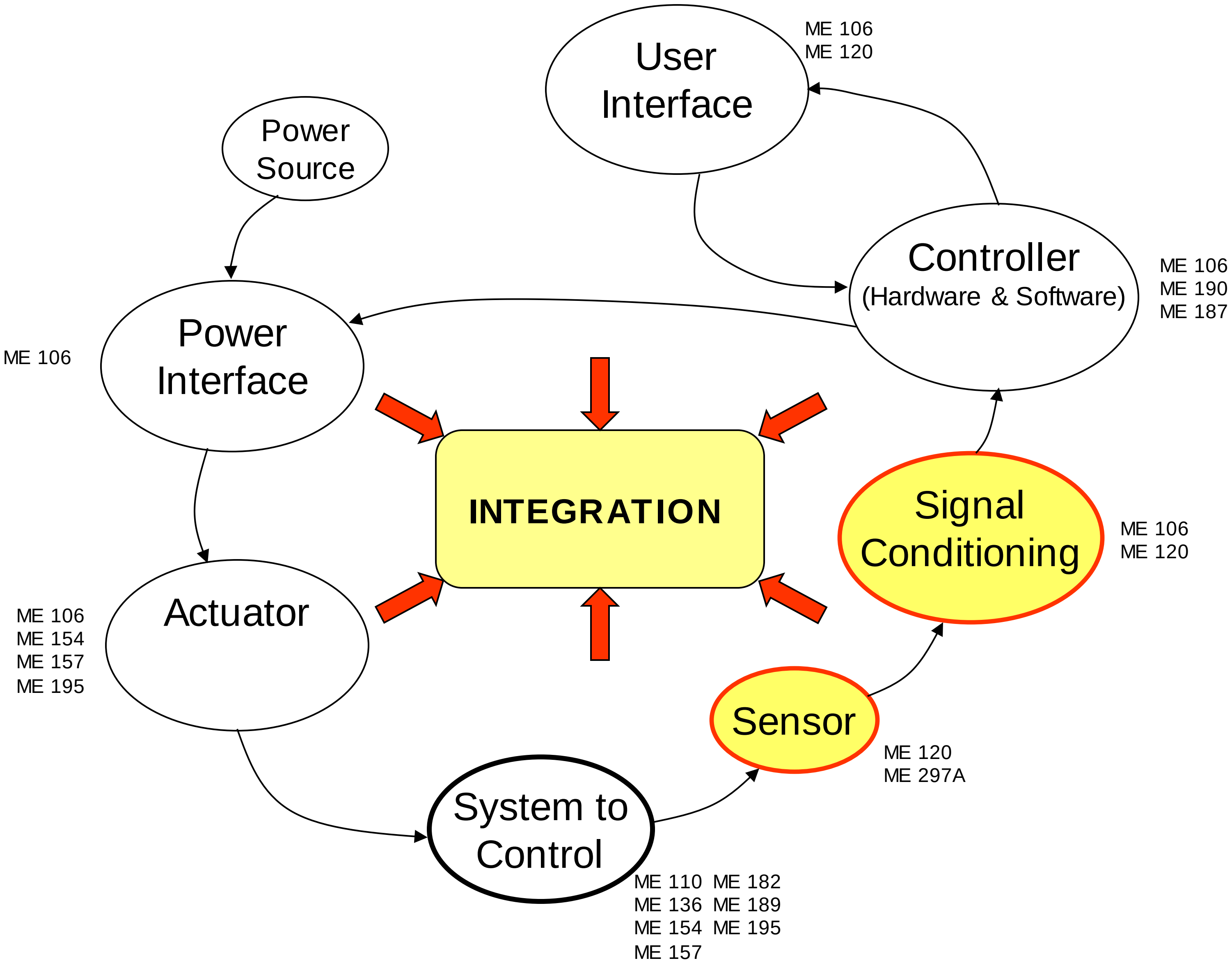
Ashby Building

Queen's University Belfast

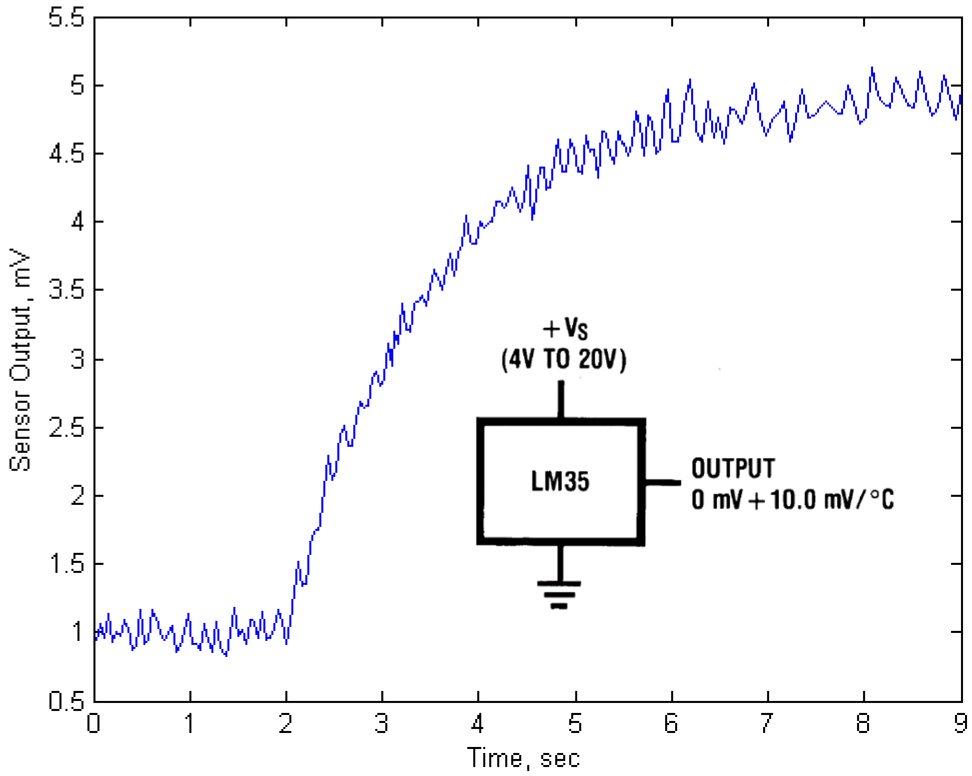
Aims

1. Filtering a noisy signal
2. High-pass and low-pass
3. Magnitude and phase angle

Mechatronics Concept Map



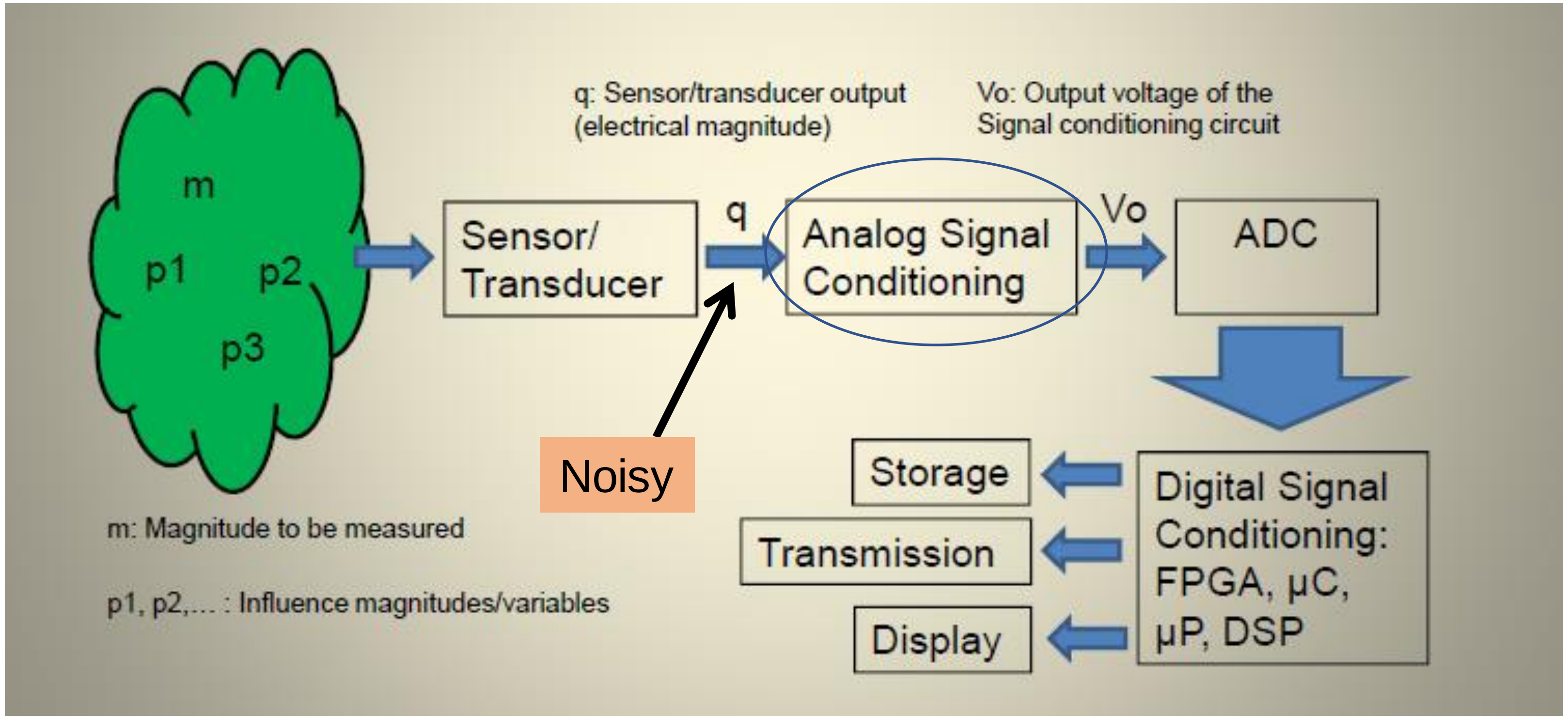
How to Handle Noisy Signals?



Filter!

Inset from: http://cache.national.com/ds/LM/LM35.pdf

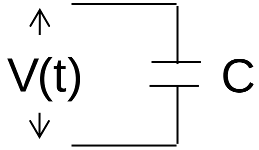
Signal conditioning



Impedance of a Capacitor

- Derive the impedance of a capacitor

$$Q(t) = CV(t) \qquad (1)$$ Physics for a capacitor



$$\text{Let } V(t) = Ae^{st} \qquad (2)$$ A time varying function

$$\frac{dQ(t)}{dt} = i(t) = C\frac{dV(t)}{dt} = CsAe^{st} \qquad (3)$$ Differentiate both sides

$$\text{Define } Z(t) = \frac{V(t)}{i(t)} \qquad (4)$$ The *impedance* – the ratio of voltage to current

$$\therefore \quad Z(t) = \frac{Ae^{st}}{CsAe^{st}} = \frac{1}{Cs} \qquad (5)$$

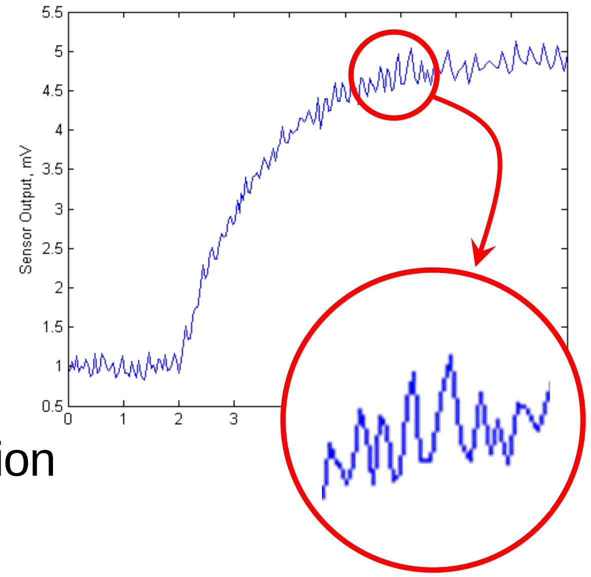
What is s?

What is s?

- Previously

$$V(t) = A\,e^{st} \qquad (2)$$

A time varying function



$$\text{Let } s = j\omega \qquad (6)$$

$$\text{where } j = \sqrt{-1}$$

$$\text{and } \omega = \text{circular frequency, in rad/s}$$

$$\therefore\ Z_C = \frac{1}{jC\omega}$$

- Why?

$$V(t) = Ae^{j\omega t} = A\left[\cos(\omega t) + j\sin(\omega t)\right] \qquad (7)$$

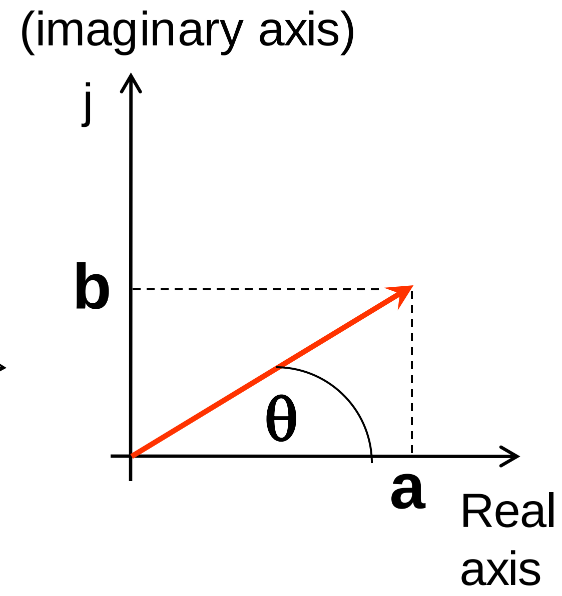
A sinusoidal function, which can also be thought of as *rotating vector*

Complex Numbers and Vectors

- Think of a complex number as a ***vector***
  - Vectors have:
    - Magnitude (length)
    - Direction (angle)

$$V(t) = Ae^{j\omega t} = A[\cos(\omega t) + j\sin(\omega t)] \qquad (7)$$

is of the form: $\boxed{a + jb}$



Magnitude = $\sqrt{a^2 + b^2}$

Theta= Direction = $\tan^{-1}\left(\frac{b}{a}\right)$

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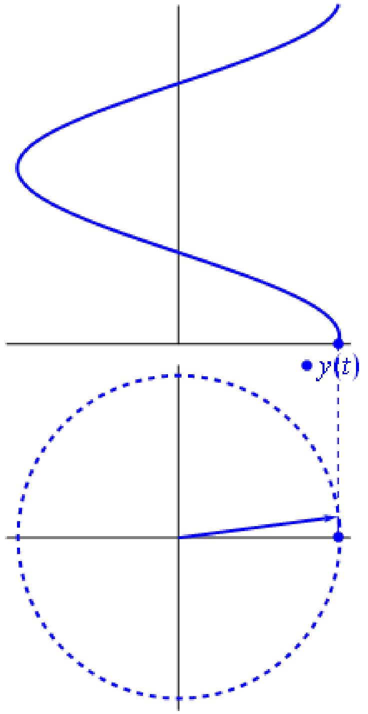
Sinusoidal Function

- Visualize the connection between the vector and the sinusoidal function of time
  - Suppose the real component is plotted as a function of time

$$V(t) = Ae^{j\omega t} = A[\cos(\omega t) + j\sin(\omega t)] \quad (7)$$

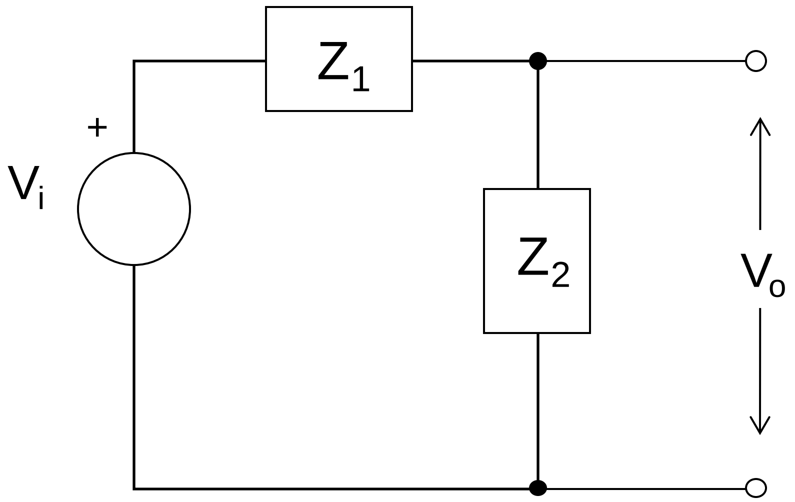
$$V_{real}(t) = A\cos(\omega t) \quad (8)$$



http://upload.wikimedia.org/wikipedia/commons/8/89/Unfasor.gif

Generalized Voltage Divider

- What is V_o in terms of V_i, Z_1, and Z_2?



$$V_O = V_I \left[\frac{Z_2}{Z_1 + Z_2} \right] \qquad (9)$$

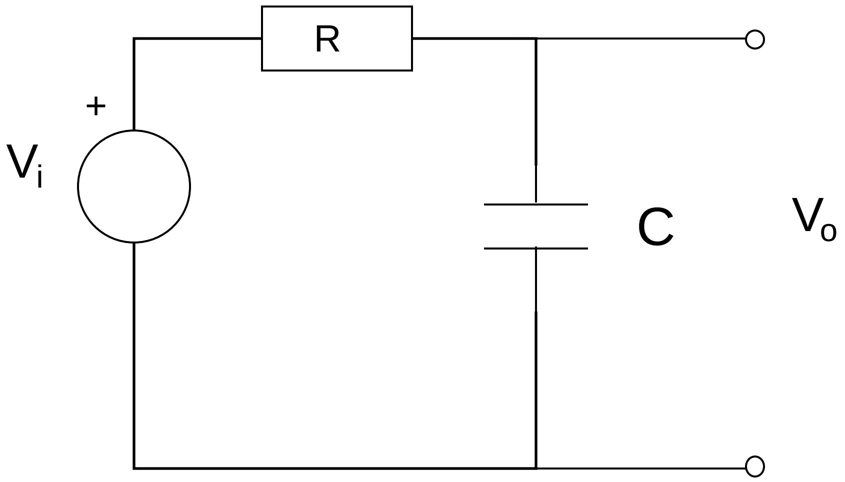
$$\frac{V_O}{V_I} = \left[\frac{Z_2}{Z_1 + Z_2} \right]$$

the 'transfer function'

RC Filters

- Frequency dependent *voltage divider*
  - Impedance of a resistor, R
  - Impedance of a capacitor, Z_C

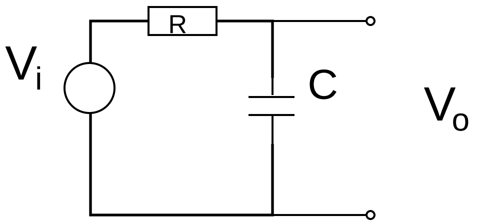
$$Z_C = \frac{1}{jC\omega} \qquad j = \sqrt{-1}$$



$$\frac{V_O}{V_I} = \left[\frac{Z_2}{Z_1 + Z_2}\right]$$

$$\frac{V_o}{V_i} = \frac{\frac{1}{jC\omega}}{\frac{1}{jC\omega} + R} = \frac{1}{1 + jRC\omega}$$

RC Filters



- Transfer function
  - Complex number

$$\frac{V_o}{V_i} = \frac{1}{1 + jRC\omega}$$

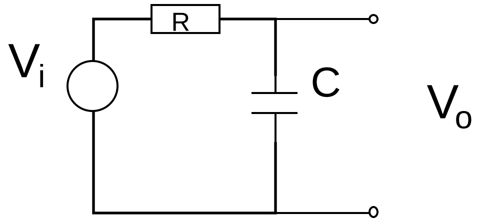
- Magnitude
  - (Magnitude of numerator)/(Magnitude of denominator)

$$\left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{1+(RC\omega)^2}}$$

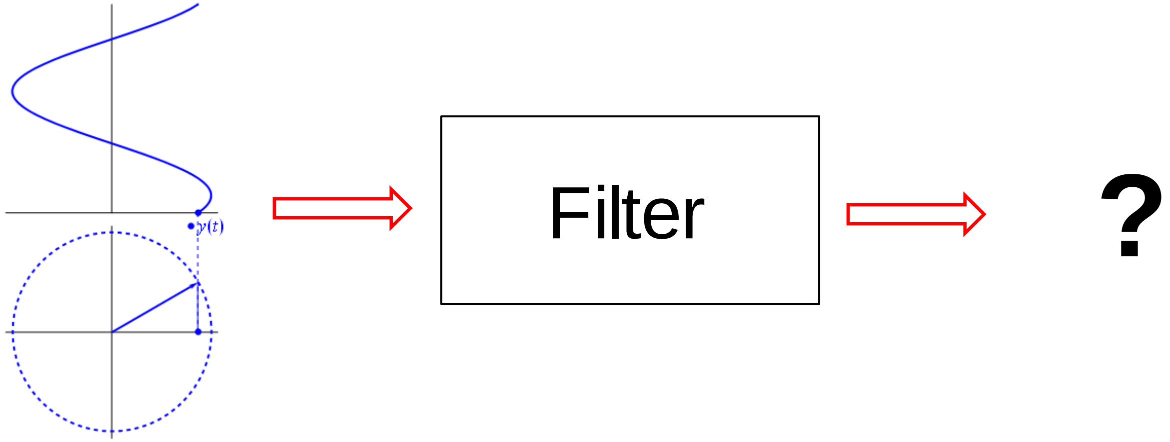
- Angle ("phase angle")
  - (angle of num) – (angle of denom)
  - How much the output is out of time synchronization with input

$$\angle \frac{V_o}{V_i} = -\tan^{-1}(RC\omega)$$

RC Filters



- Behavior
  - How does the magnitude and angle of the transfer function change *with frequency*?
    - Ex. R=10k, C=1uF

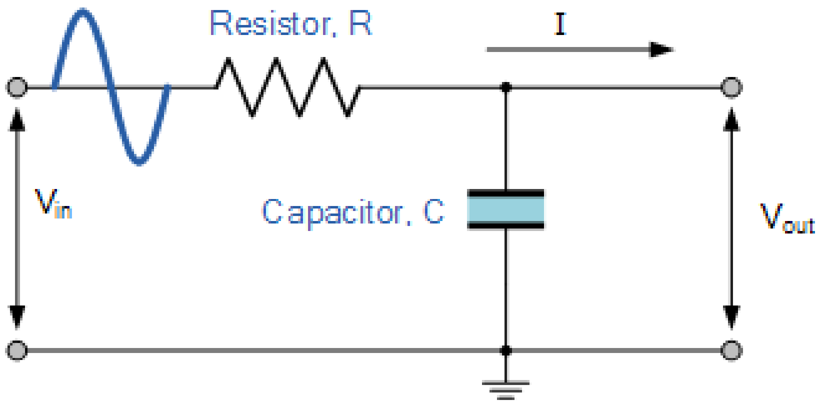


FILTERING

- The three most common types of filters are called
  - Butterworth
    - fairly flat response in the pass-band
    - a steep attenuation rate,
    - a non-linear phase response
  - Chebyshev
    - steeper rate of attenuation, develop some ripple in the pass band.
    - The phase response is much more non-linear than the Butterworth.
  - Bessel
    - have the best step response and phase linearity. But requires a number of stages or orders

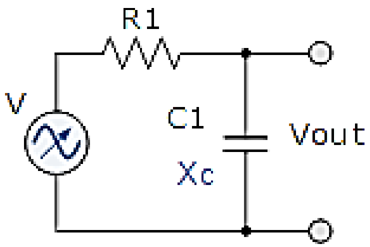
LOW-PASS FILTER

- A low-pass filter allows for easy passage of low-frequency signals from source to load, and difficult passage of high-frequency signals.
- The cutoff frequency for a low-pass filter is that frequency at which the output (load) voltage equals 70.7% of the input (source) voltage.
- Above the cutoff frequency, the output voltage is lower than 70.7% of the input, and vice versa.

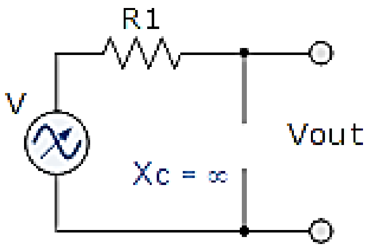


$$X_C = \frac{1}{2\pi f C}$$

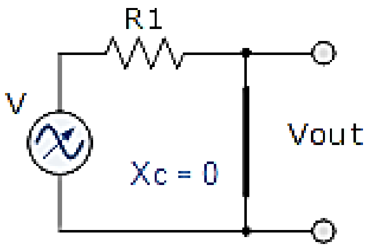
First order low pass filter



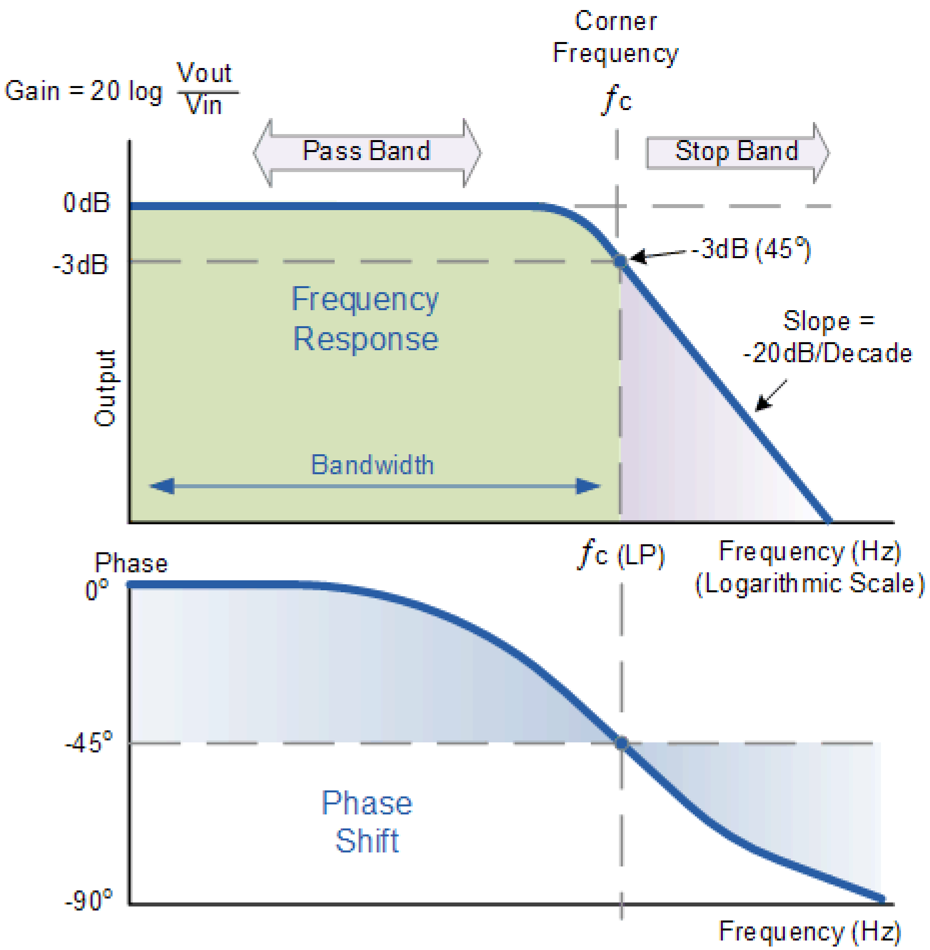
Low Pass at normal frequency



Low Pass at DC zero frequency



Low Pass at high frequency



Cut-off Frequency and Phase Shift

$$fc = \frac{1}{2\pi RC}$$

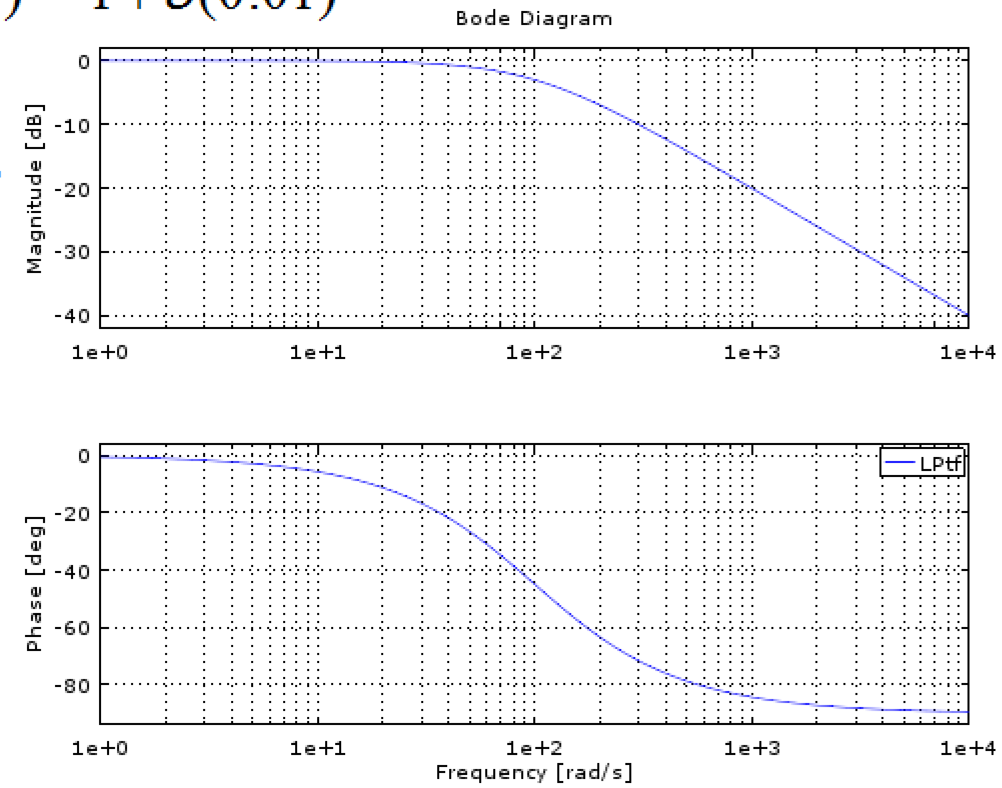
$$\text{Phase Shift } \varphi = -\arctan(2\pi f RC)$$

Bode Plots – Matlab/Octave Style

$$\frac{V_o}{V_i} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + sRC}$$

$$= \frac{1}{1 + s(1\times10^4\,\Omega)(1\times10^{-6}\,F)} = \frac{1}{1 + s(0.01)}$$

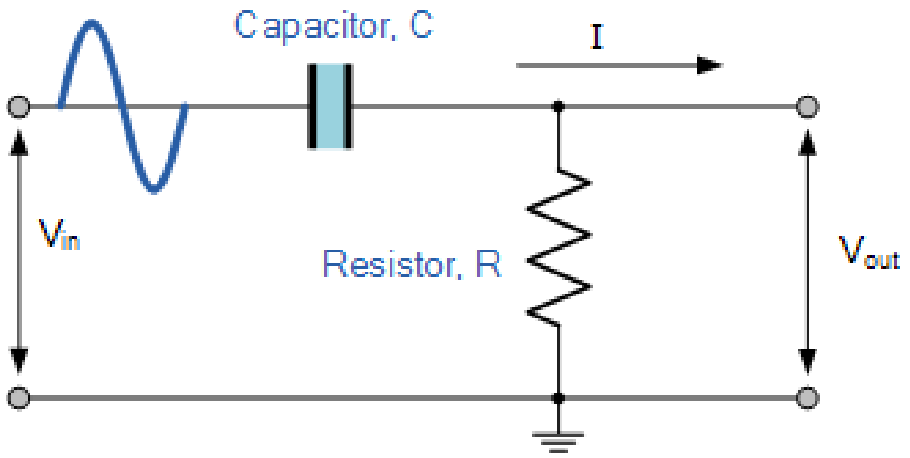
```
>> s = tf('s');
>> LPtf = 1/(1+s*0.01);
>> bode(LPtf)
```



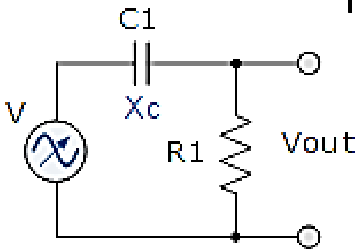
HIGH-PASS FILTER

A high-pass filter allows for easy passage of high-frequency signals from source to load, and difficult passage of low-frequency signals.

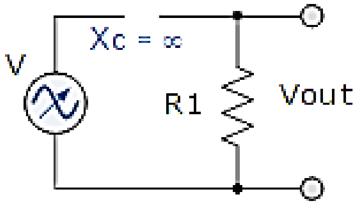
The cutoff frequency for a high-pass filter is that frequency at which the output (load) voltage equals 70.7% of the input (source) voltage. Above the cutoff frequency, the output voltage is greater than 70.7% of the input, and vice versa.



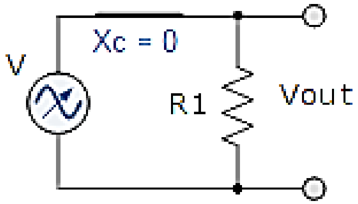
First order high pass filter



High Pass at normal frequency



High Pass at DC zero frequency



High Pass at high frequency

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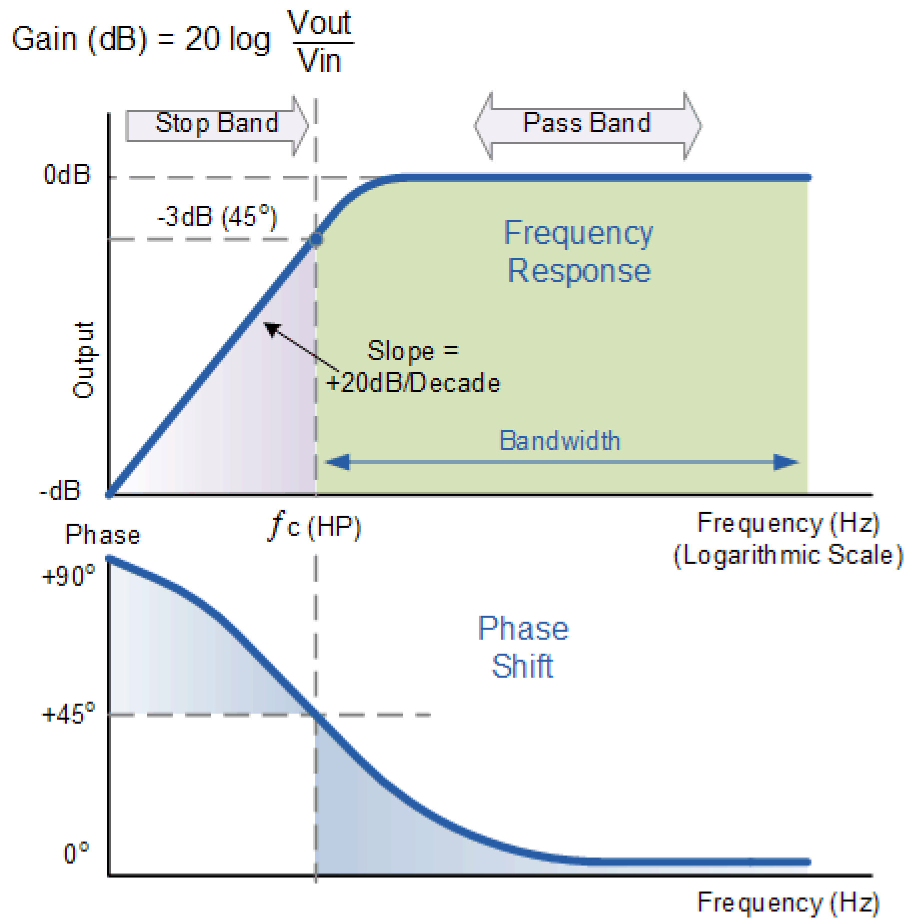


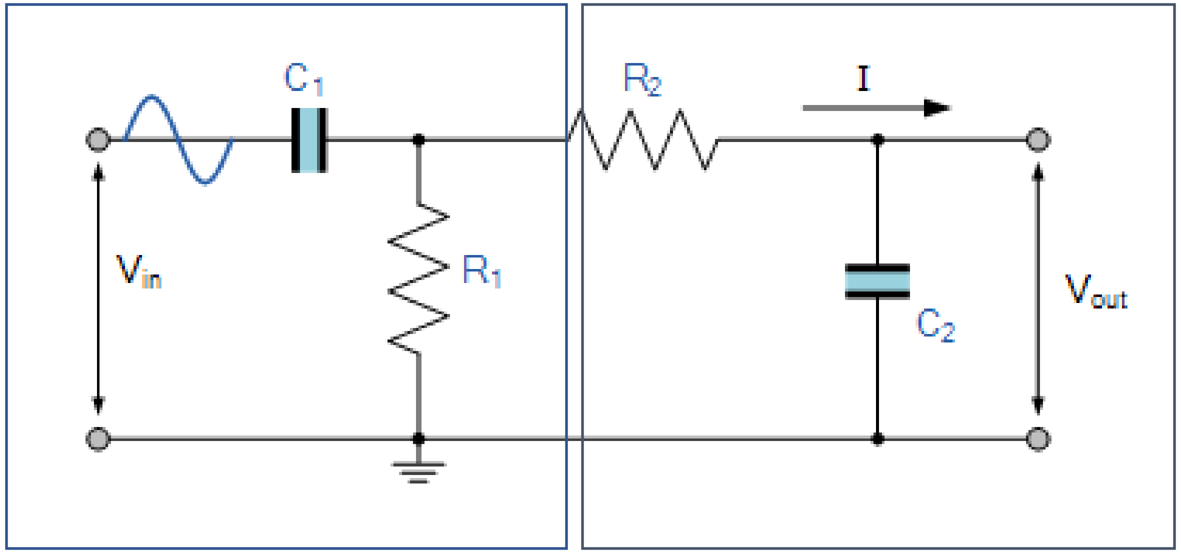
Cut-off Frequency and Phase Shift

$$f_c = \frac{1}{2\pi RC}$$

$$\text{Phase Shift } \varphi = -\arctan(2\pi f RC)$$

BAND-PASS FILTER

- By connecting or “cascading” together a single High Pass Filter circuit with a Low Pass Filter circuit, we can produce another type of passive RC filter that passes a selected range or “band” of frequencies that can be either narrow or wide while attenuating all those outside of this range.
- known commonly as a Band Pass Filter



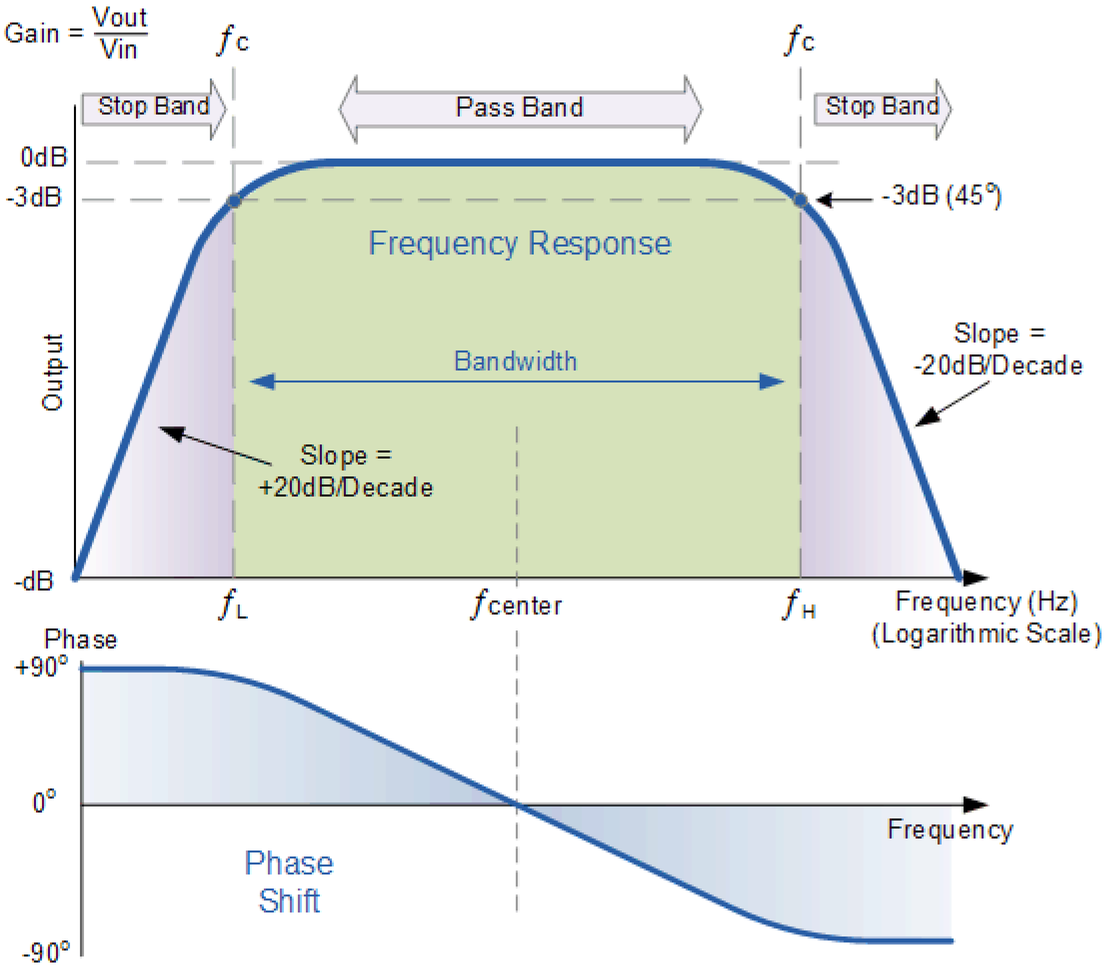
Cut-off Frequency and Phase Shift

$$fc = \frac{1}{2\pi RC}$$

Phase Shift $\varphi = -\arctan(2\pi f RC)$

To set the first band pass frequency (HP)

To set the second band pass frequency (LP)



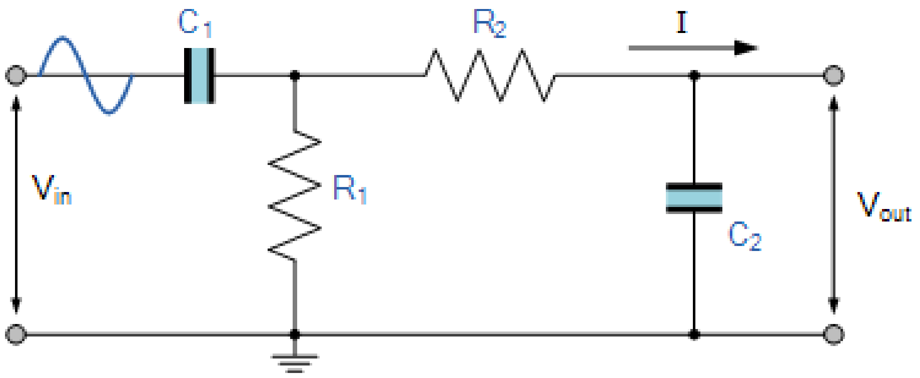
Cut-off Frequency and Phase Shift

$$f_c = \frac{1}{2\pi RC}$$

$$\text{Phase Shift } \varphi = -\arctan(2\pi f RC)$$

EXAMPLE 1

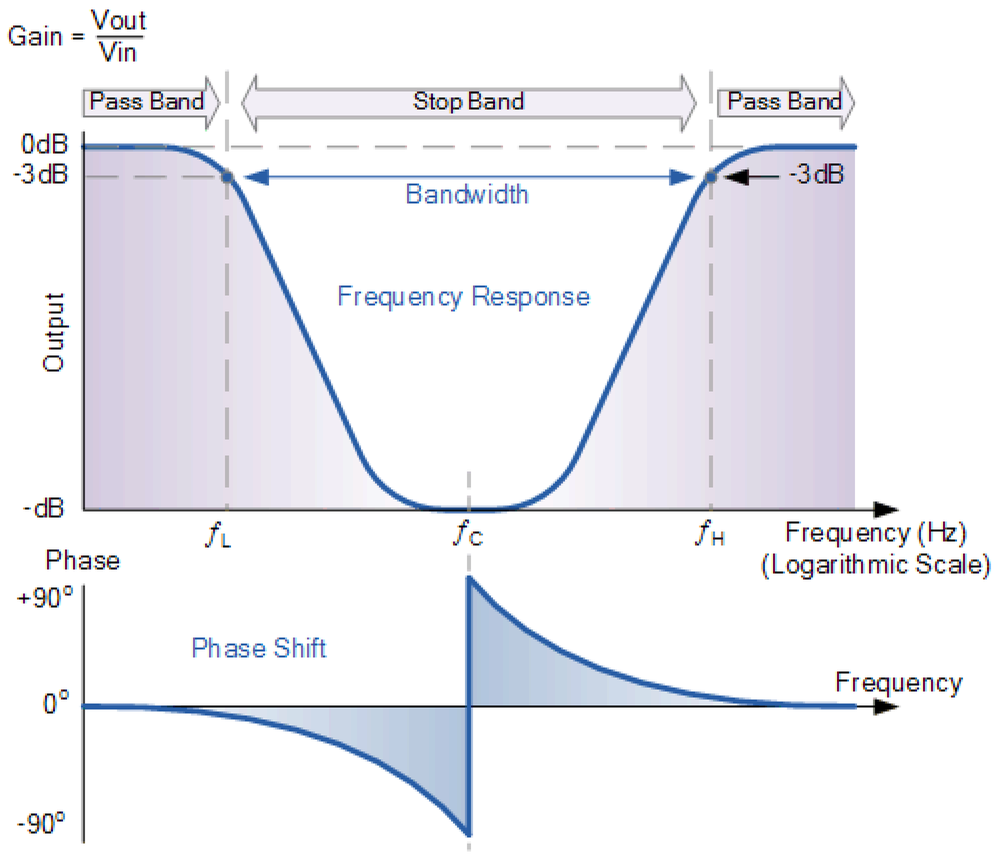
A second-order band pass filter is to be constructed using RC components that will only allow a range of frequencies to pass above 1kHz (1,000Hz) and below 30kHz (30,000Hz). Assuming that both the resistors have values of 10kΩ´s, calculate the values of the two capacitors required



Answers:
C_1 = 530 pF
C_2 = 15.8 nF

BAND-STOP FILTER

- combine the low and high pass filter to produce another kind of RC filter network
- that can block or at least severely attenuate a band of frequencies within these two cut-off frequency points.



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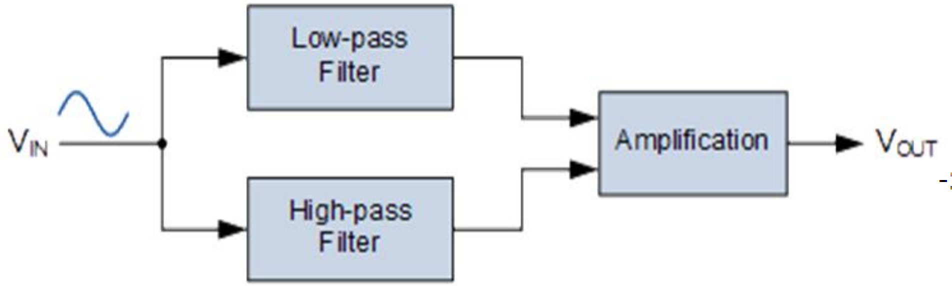
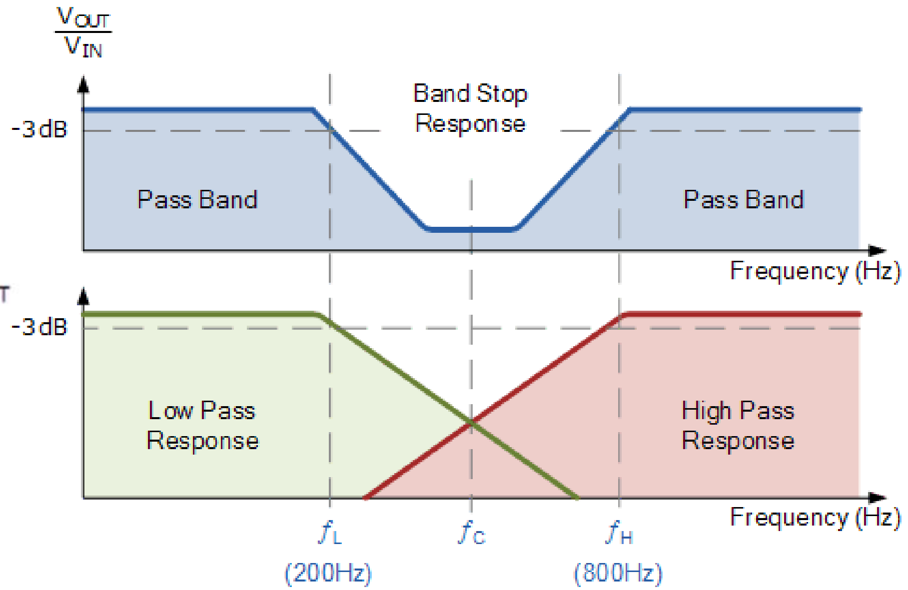
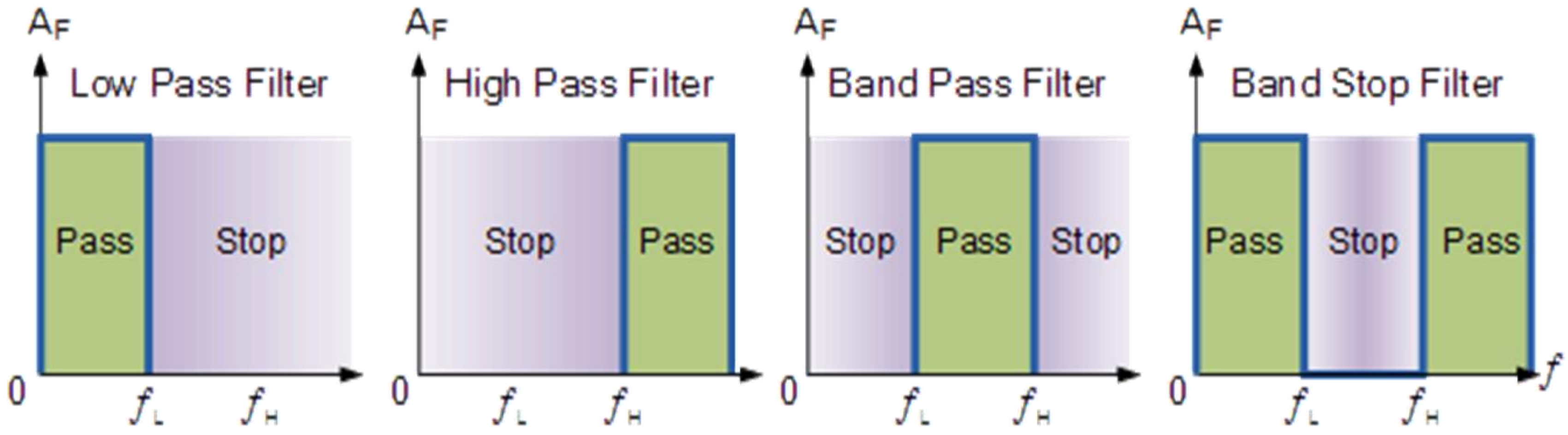
- Band-pass filters are constructed by combining a low pass filter in series with a high pass filter
- Band stop filters are created by combining together the low pass and high pass filter sections in a “parallel” type configuration as shown.

Cutoff frequency is 200Hz

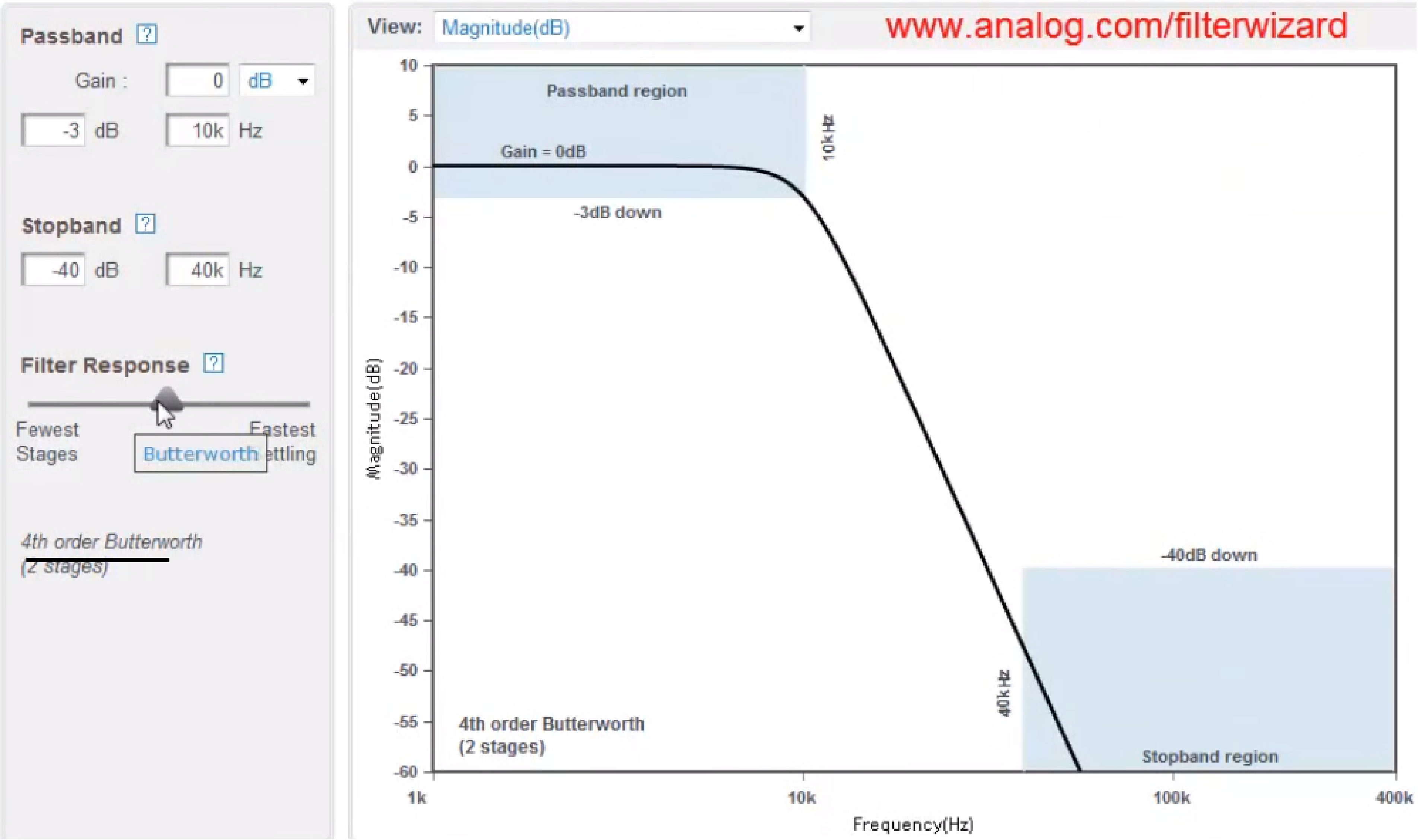


Cutoff frequency is 800Hz

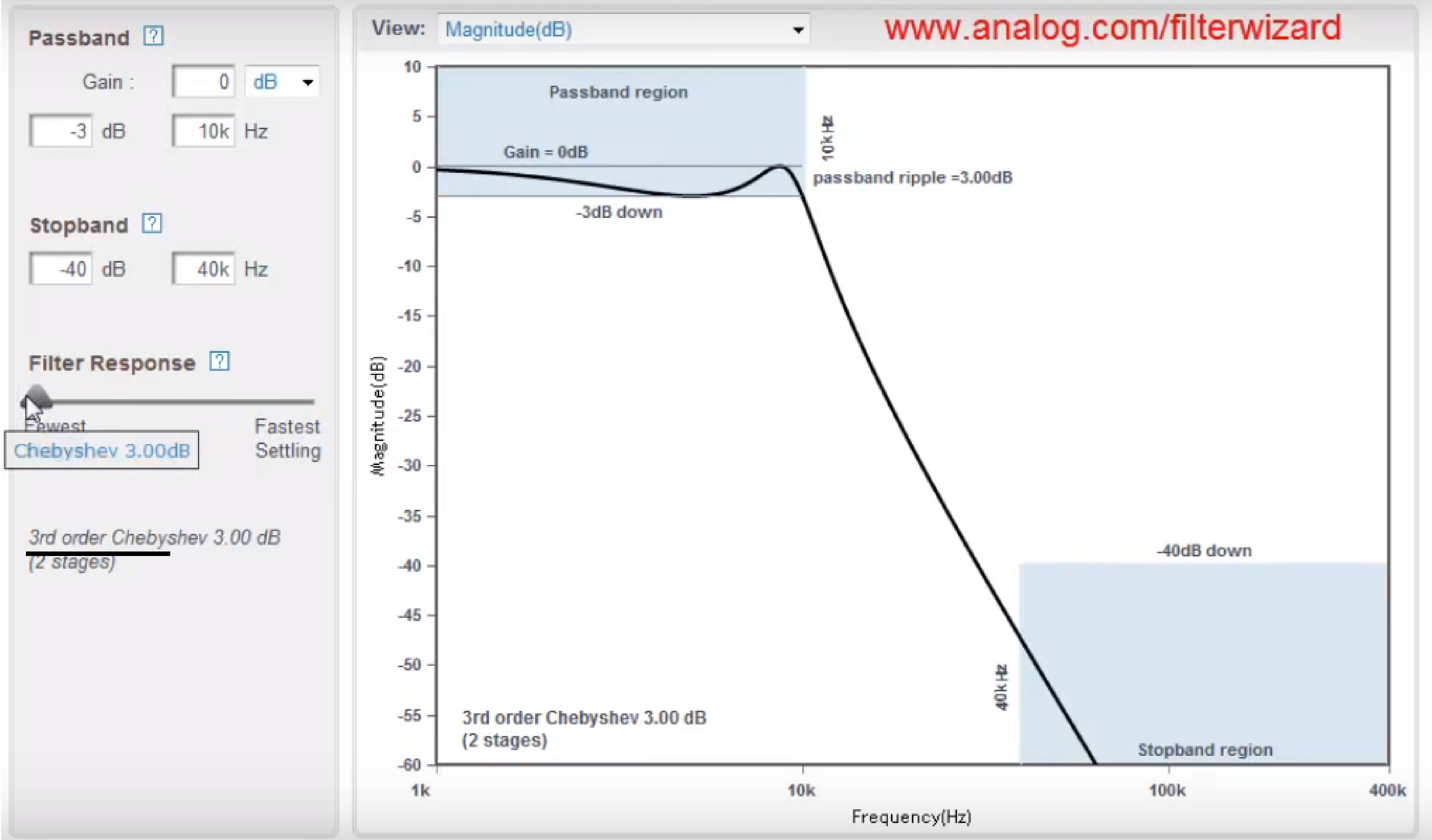


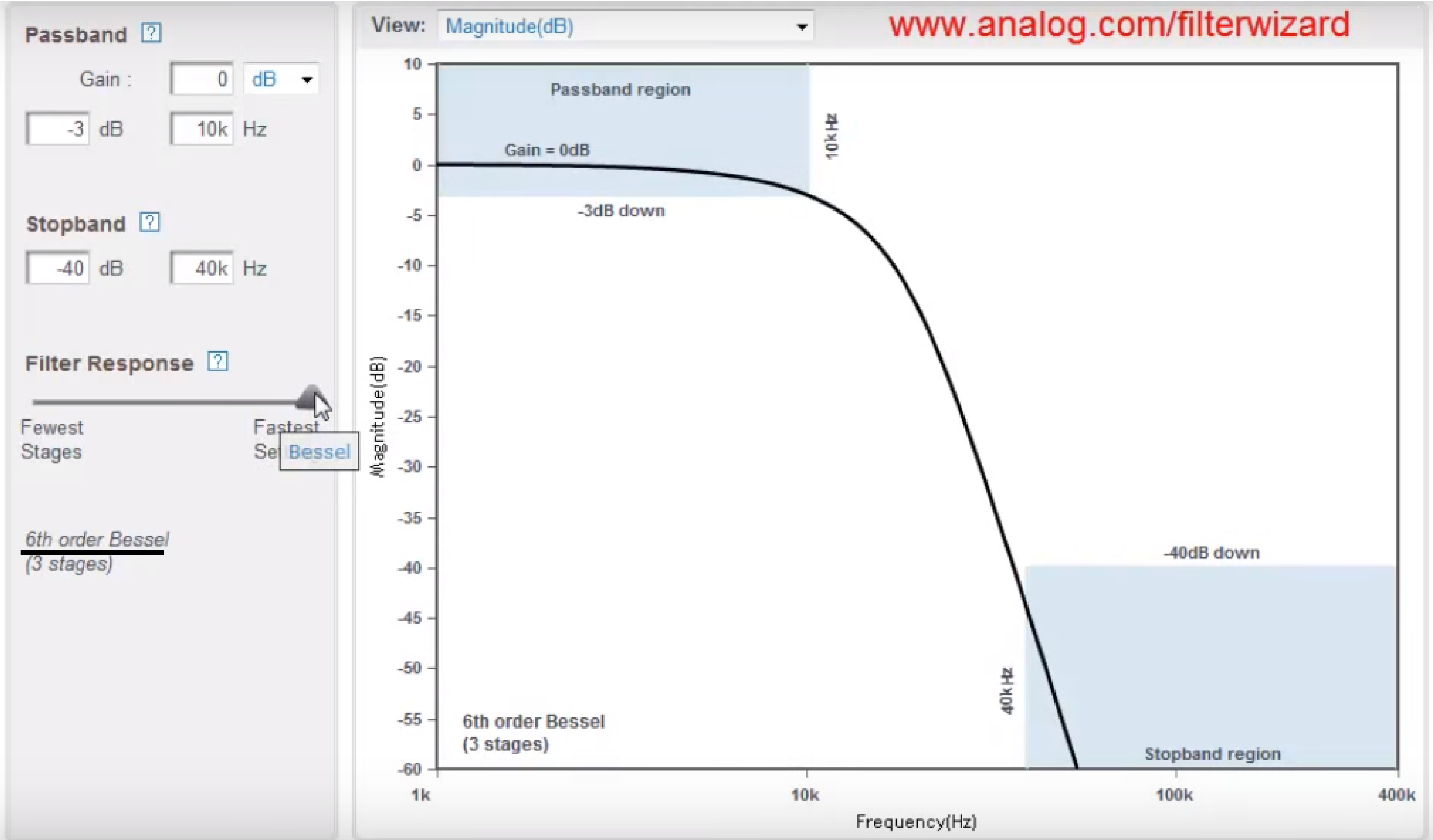


AF
Low Pass Filter
Pass
Stop
0
fL
fH
AF
High Pass Filter
Stop
Pass
0
fL
fH
AF
Band Pass Filter
Stop
Pass
Stop
0
fL
fH
AF
Band Stop Filter
Pass
Stop
Pass
0
fL
fH
f

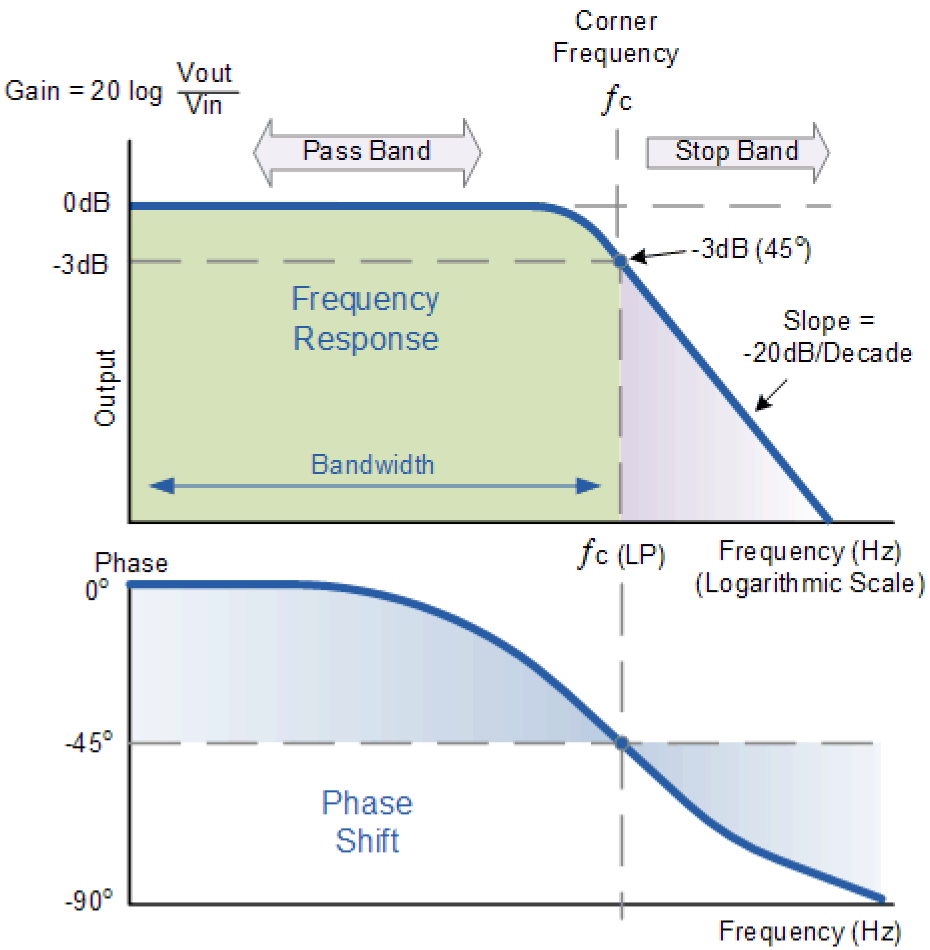


Passband
Gain : 0 dB
-3 dB 10k Hz
Stopband
-40 dB 40k Hz
Filter Response
Fewest Stages
Fastest Settling
Butterworth
4th order Butterworth (2 stages)
View: Magnitude(dB)
www.analog.com/filterwizard
Passband region
Gain = 0dB
-3dB down
10k Hz
40k Hz
-40dB down
Stopband region
4th order Butterworth (2 stages)
Magnitude(dB)
Frequency(Hz)
1k
10k
100k
400k


Passband
Gain : 0 dB
-3 dB 10k Hz
Stopband
-40 dB 40k Hz
Filter Response
Fewest
Chebyshev 3.00dB
Fastest Settling
3rd order Chebyshev 3.00 dB (2 stages)
View: Magnitude(dB)
www.analog.com/filterwizard
Passband region
Gain = 0dB
10k Hz
passband ripple =3.00dB
-3dB down
-40dB down
40k Hz
3rd order Chebyshev 3.00 dB (2 stages)
Stopband region
Magnitude(dB)
Frequency(Hz)
1k
10k
100k
400k



Passband
Gain : 0 dB
-3 dB 10k Hz
Stopband
-40 dB 40k Hz
Filter Response
Fewest Stages
Fastest Se
Bessel
6th order Bessel
(3 stages)
View: Magnitude(dB)
www.analog.com/filterwizard
Passband region
Gain = 0dB
-3dB down
10k Hz
-40dB down
40k Hz
6th order Bessel
(3 stages)
Stopband region
Magnitude(dB)
Frequency(Hz)
1k
10k
100k
400k



At low frequency (Pass Band)

$$\frac{V_{out}}{V_i} = Gain, Av$$

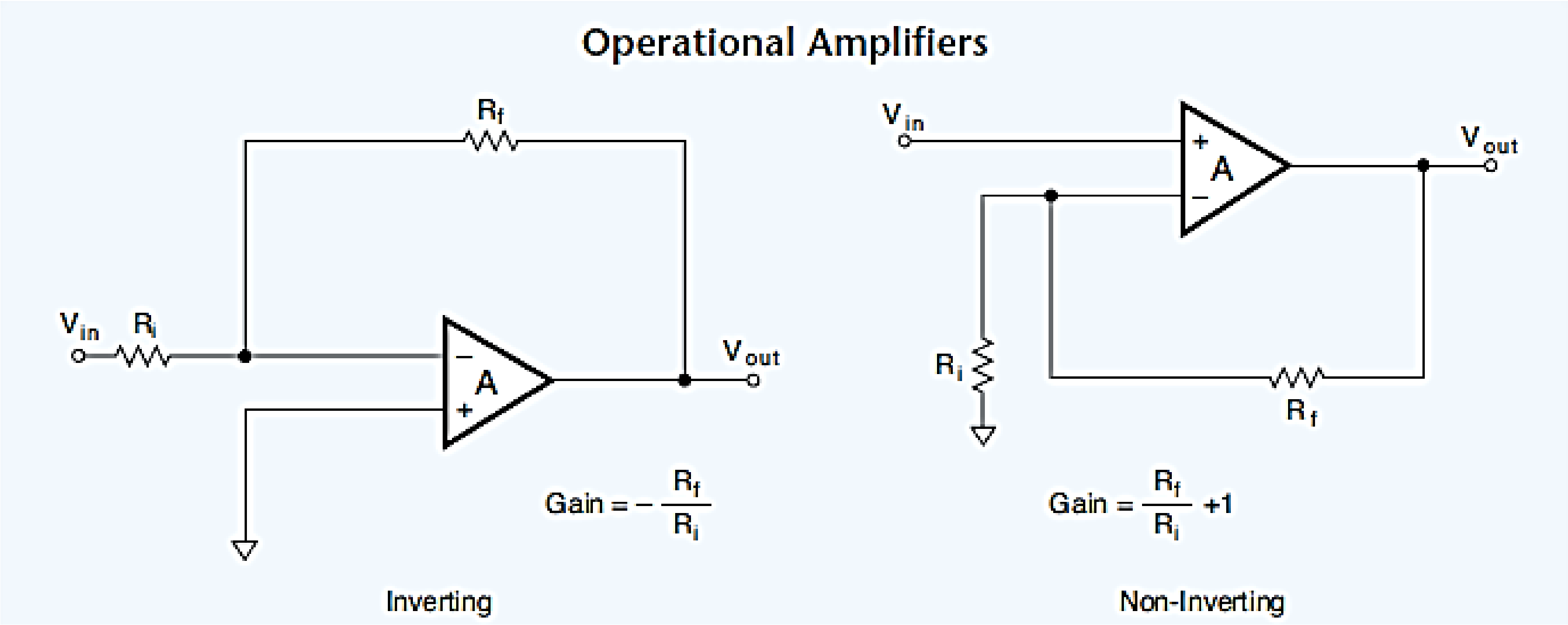
At cutoff frequency

$$\frac{V_{out}}{V_i} = 0.7071\ Av$$

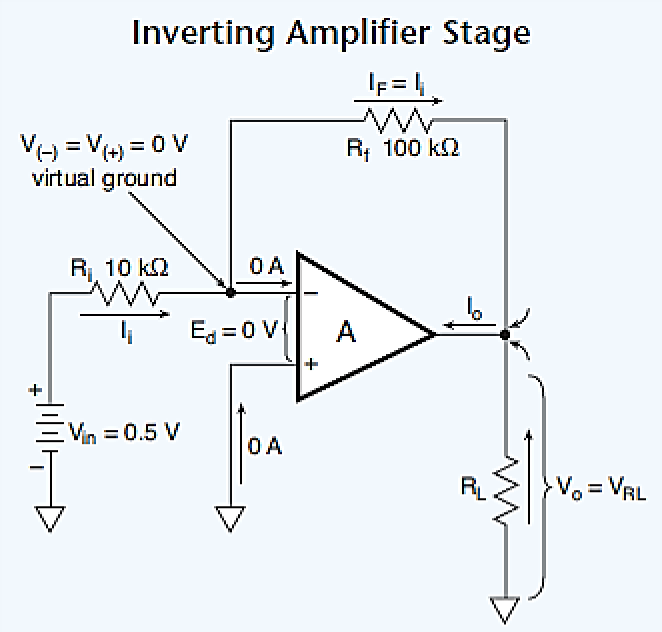
AMPLIFICATION

- Many sensors develop extremely low-level output signals
- Two common examples of low-level sensors are thermocouples and strain-gauge (less than 50 mV)
- Requires amplification
  - Using Operational Amplifier (Op-Amp)
    - Inverting Amplifier
    - Non-Inverting Amplifier
    - Differential Amplifier
    - Instrumentation Amplifier

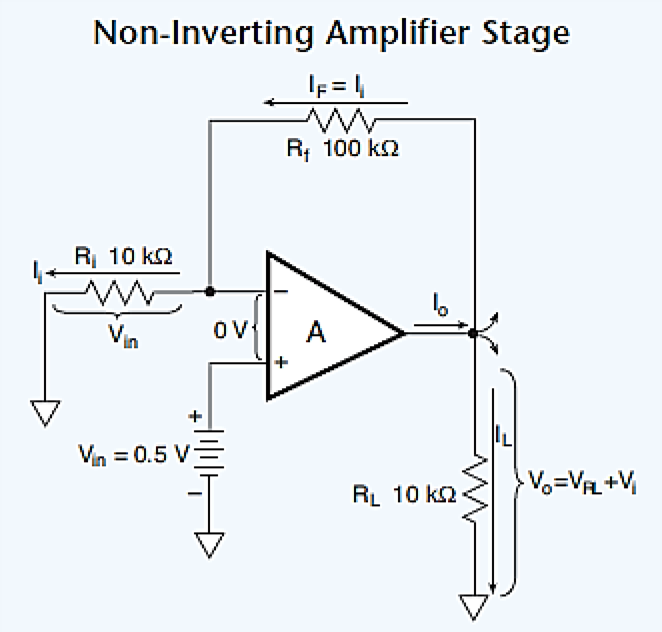
INVERTING AND NON-INVERTING AMPLIFIERS



NOTE: The maximum input signal that the amplifier can handle without damage is usually about 2 V less than the supply voltage. For example, when the supply is ±15 V_{DC}, the input signal should not exceed ±13 V_{DC}.

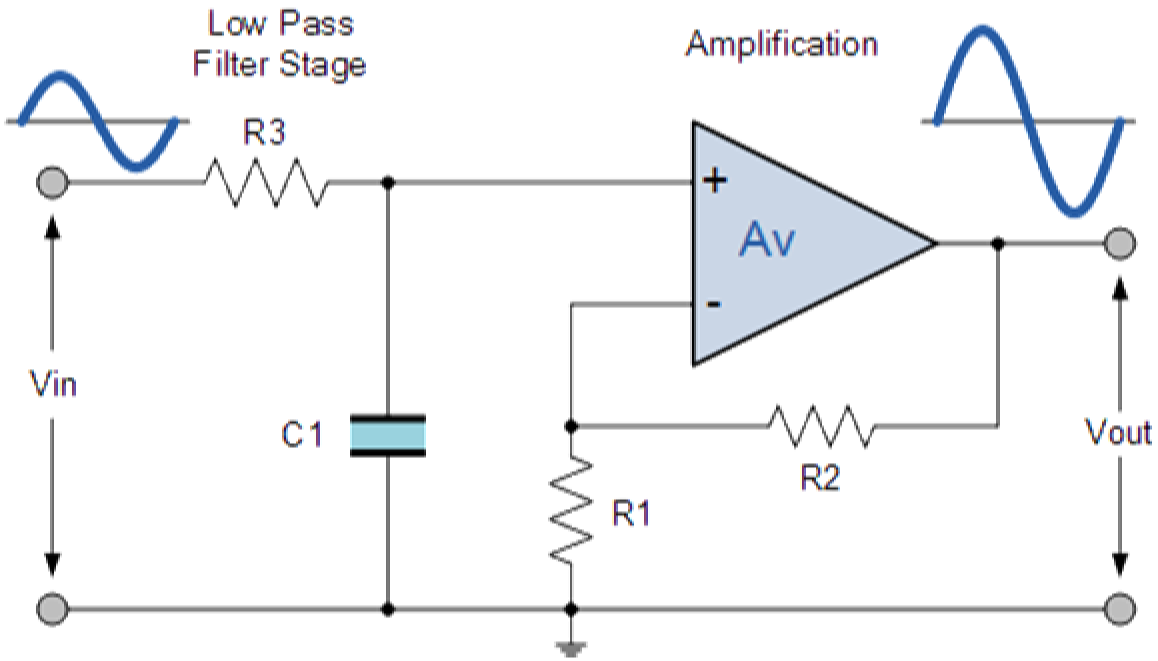


Gain = -10
Vo = - 5 V



Gain = 11
Vo = 5.5 V

- Passive filters use R, L and C while Active Filters use R, L and C with combination of an active component such as Op-Amp.
- Main disadvantage of passive filters is that the amplitude of the output signal is less than that of the input signal



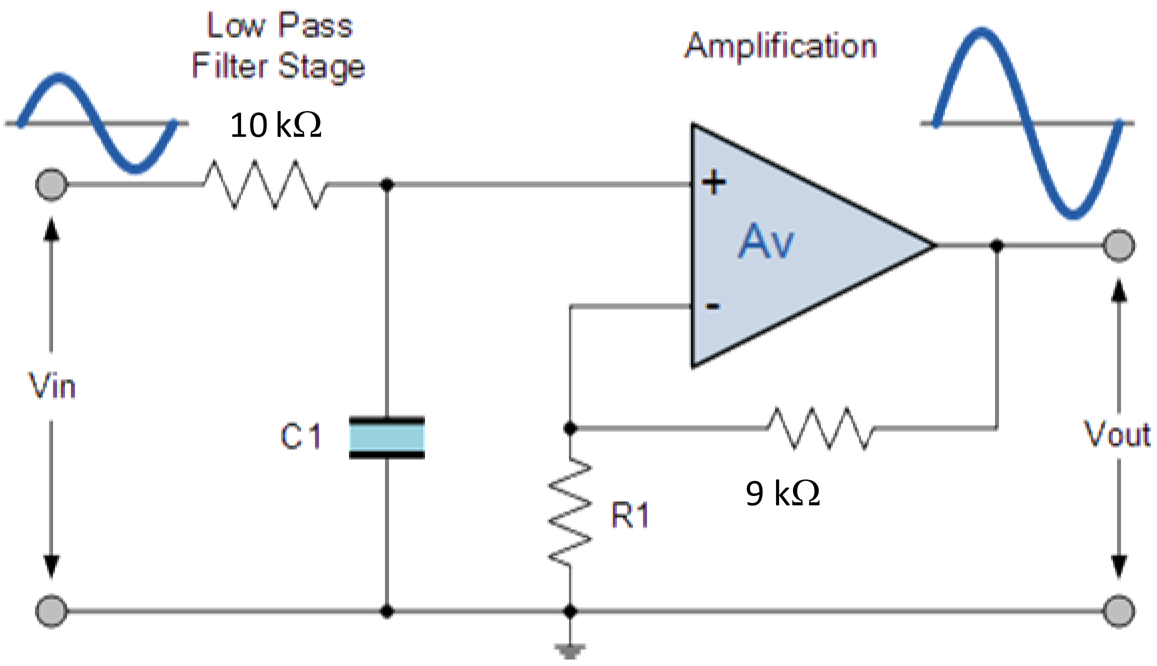
at low f : $X_C \to \infty$, $V_{out} = V_{in}$

at high f : $X_C \to 0$, $V_{out} = 0$

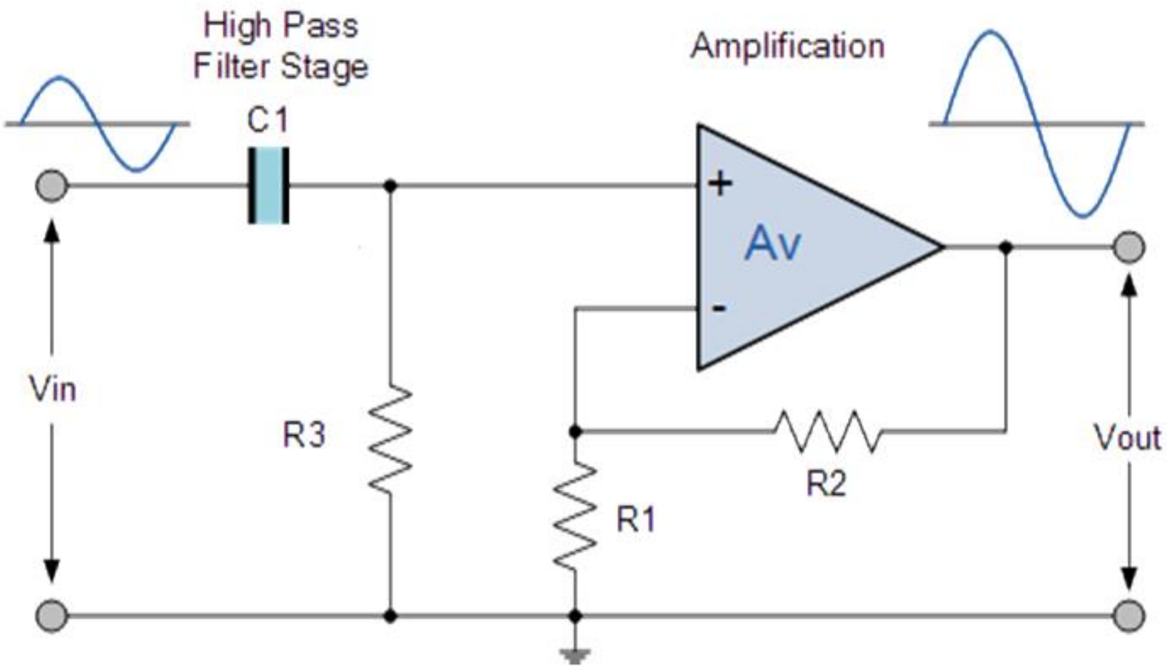
NON - INVERTING CONFIGURATION

Gain, $A_V = 1 + \frac{R_2}{R_1}$

- Design a non-inverting active low pass filter circuit that has a gain of ten at low frequencies, given that the resistor of the filter is 10 kΩ , the feedback resistor is 9 kΩ and a high frequency cut-off or corner frequency of 159 Hz



R_1 = 1 kΩ
C_1 = 100 nF

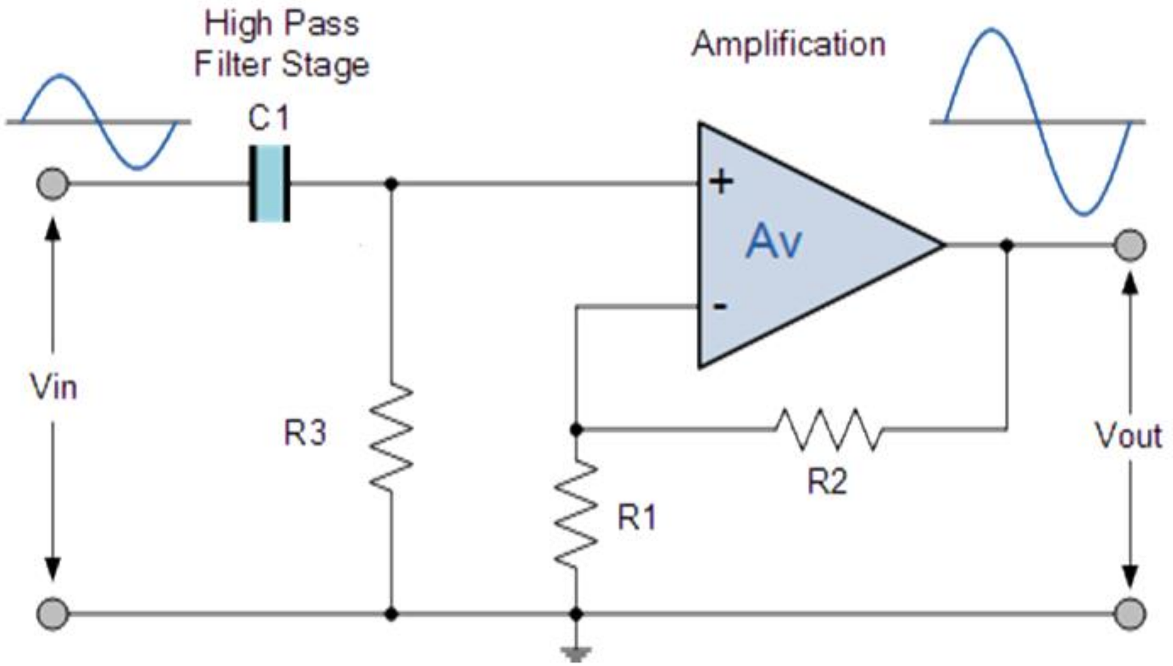


NON - INVERTING CONFIGURATION

$$\text{at low } f: \ Xc \to \infty, \ \ Vout = 0$$
$$\text{at high } f: \ Xc \to 0, \ \ Vout = Vin$$

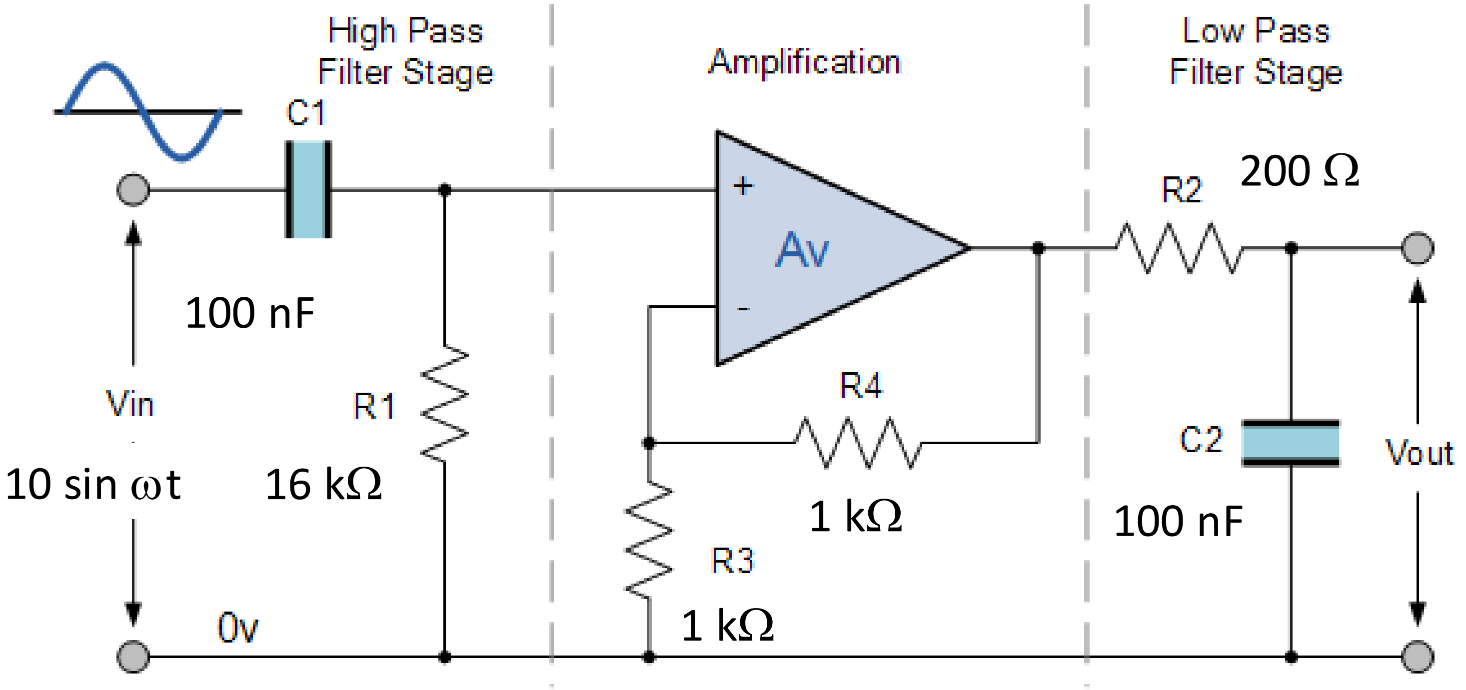
Gain, $A_V = 1 + \dfrac{R_2}{R_1}$

A first order active high pass filter has a pass band gain of two and a cut-off corner frequency of 1 kHz. If the input capacitor has a value of 10 nF, calculate the value of the resistor of the filter and the resistors of the amplifier.

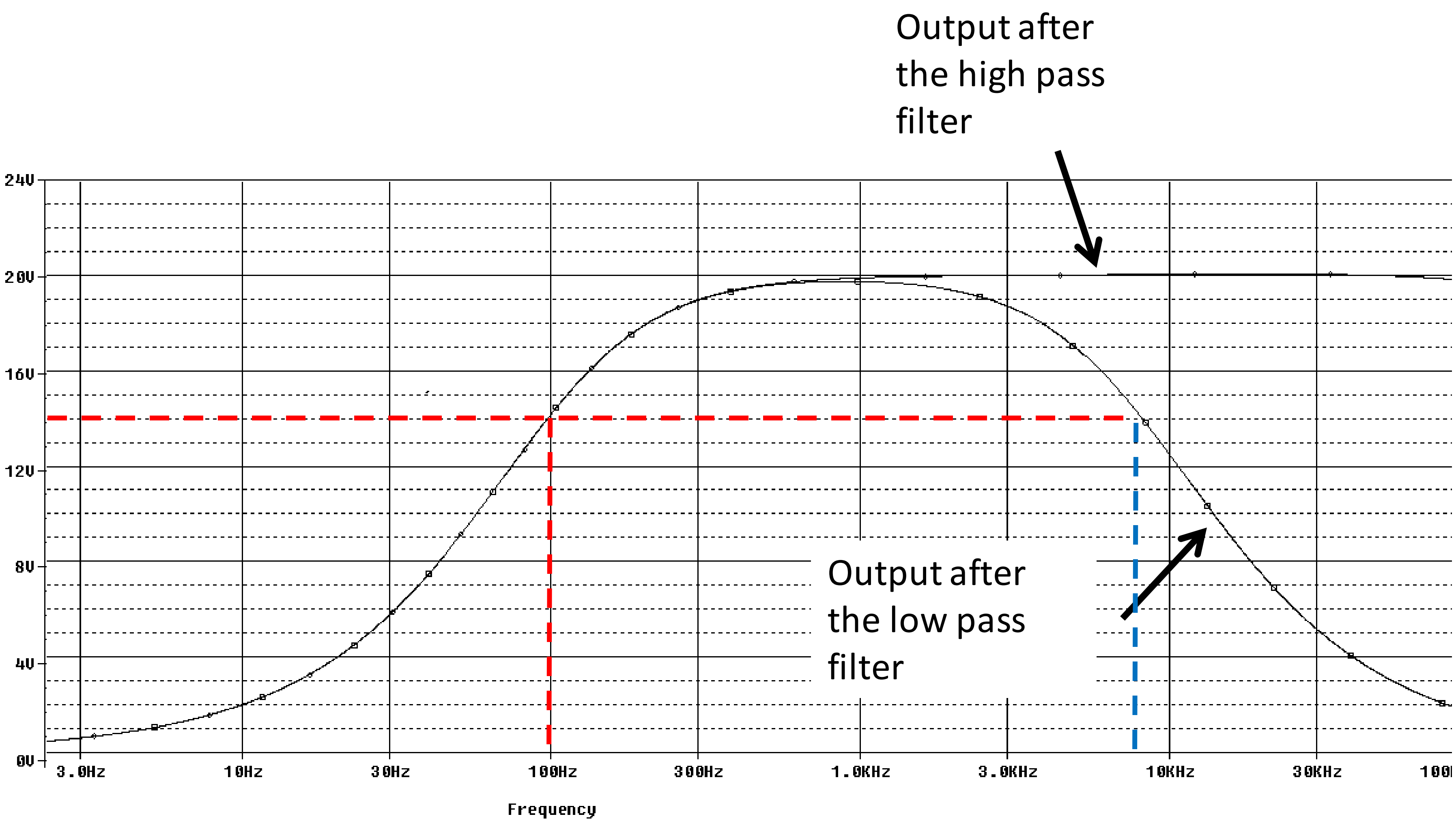


$R_3 = 16\ k\Omega$
$R_1 = R_2$

BAND PASS - ACTIVE FILTER



NON - INVERTING CONFIGURATION


Output after
the high pass
filter
24V
20V
16V
12V
8V
4V
0V
3.0Hz
10Hz
30Hz
100Hz
300Hz
1.0KHz
3.0KHz
10KHz
30KHz
Frequency
Output after
the low pass
filter

Thank You For Your Attention!

Any Question?

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